Wilby CE (VA) Primary School

Our whole school approach to calculation 2022

Introduction

The purpose of this booklet is to outline the stages of calculation from EYFS to year 6 using the four operations (+ - x ÷). Written methods of calculation require the use of mental strategies and build on informal jottings and informal written methods of recording as well as a deep understanding of place value and a conceptual understanding of number. These methods of calculation will only be introduced in EYFS or key stage 1 when children are secure in the number skills set out below.

- Counting to 20
- 1:1 counting of objects by moving them
- 1:1 counting of actions / things that cannot be moved, including in a random order (see 5 principles of counting from the counting assessment)
- Putting fingers out when counting - each finger representing one thing
- Knowing Numicon plates (coloured and grey) and subtraction covers, representing the structure with other objects eg. pegs, counters
- Structure of numbers to 10 and 20 using Rekenrek
- Counting in 10s from 0
- Counting in 10s from any number
- Year 1s – writing numbers to 99 / place value of numbers to 99
- By end of reception – be secure in number bonds to 10
- Year 1s – be secure in number bonds to 20

Children will be encouraged to use mental strategies as a first resort. Children will be taught and given opportunities to use and apply a wide variety of mental calculations alongside the written strategies. They will be encouraged to **use mental strategies as a first resort** and will need to consider the most appropriate strategy for the numbers involved. It is **not** recommended that children learn a written calculation method with smaller/easier numbers first as they should not be using a written method to if the numbers are able to be calculated mentally.

Children will use the written calculation method they are secure with, irrespective of their year group, when problem solving or carrying out standalone calculations. They will be working towards the next stage in the development of their calculation skills during in lessons specifically planned to develop these. **Although we follow the White Rose Hub schemes, we are still to follow this policy where calculation strategies are to be taught.**

Children will be encouraged to continue to use a secure strategy alongside a new strategy until they are confident enough in using the new strategy.

Progression in all calculation will follow this structure and will be, wherever possible, taught in context:

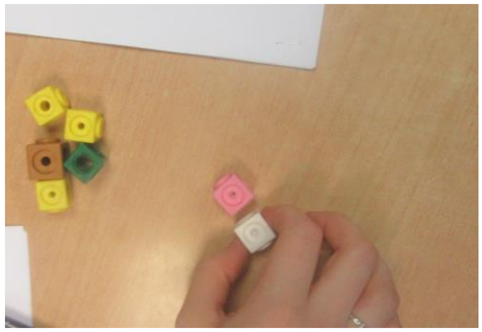
- practically/ with concrete apparatus
- symbolically/ pictorially
- using informal jottings
- using number lines
- horizontal calculations
- vertical calculations

The strategies within this booklet have been organised in the order in which our staff agreed they should be taught. It is really important that teachers and parents take into account the level of understanding their children are working at so that children have a sound understanding of the mathematics involved in a strategy, and are not just learning 'rules without reasons'. Previous stages may need to be revisited to consolidate understanding where necessary.

All children by the end of year 1, should have, at their level, a reliable method for calculating with all four operations which they understand and can explain. From Year 1 to Year 6, children should have their own calculation booklet, which they can refer to, showing their method for each of the 4 operations.

Addition

• Children will put 2 groups together and count the total. 1:1 counting will be used, with children moving or touching each object as they count.



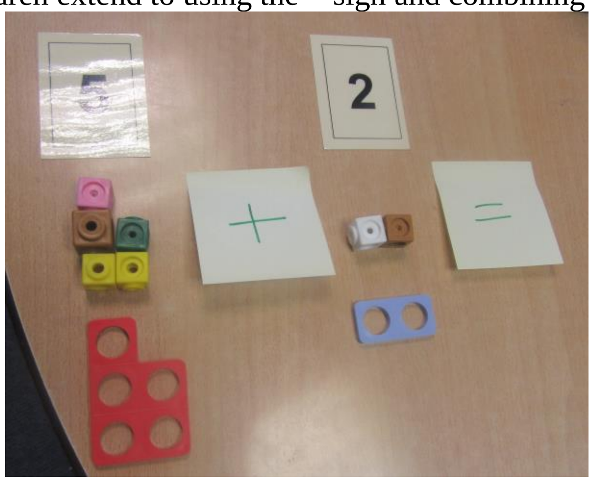
• They will add 1 more to a group. Firstly with numbers up to 5, then to 10. 1:1 counting will be used, with children moving each object as they count.

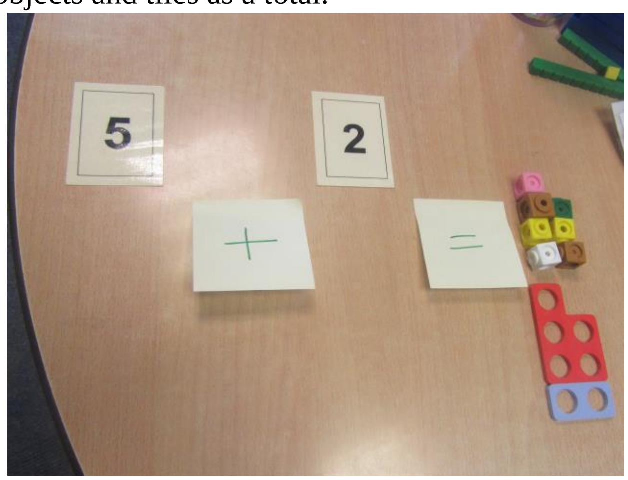
• Children will organise objects into the Numicon structure so the amounts can be subitised rather than using 1:1 counting to find the total. Children will have 2 groups (that will total below 10) and combine to find the total (using objects and tiles).

• Match groups of objects to tiles and digit cards.

• Children will include the + symbol into their calculation which uses objects, Numicon tiles and digit cards (initially adding 2 single digits then 3 single digits)

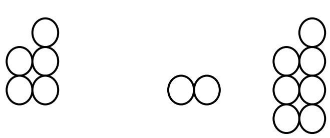
• Children extend to using the = sign and combining objects and tiles as a total.





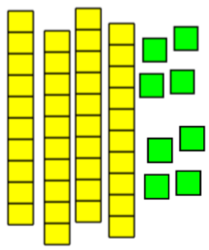
• Children will record calculations using drawings of Numicon plates with + and = symbols (Drawing 1s as circles which will lead later into Base Ten drawings)

5 + 2 = 7



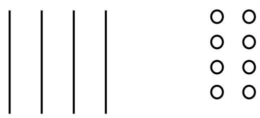
• They will add a single digit to 10 following the same methods as above.
• Children will add to make number bonds to 20.

• Children will be adding larger numbers so will use Base Ten to physically make calculations eg. 40+8, setting out ones in the Numicon structure (begin with no going over the tens boundary).



N.B. Our Base Ten blocks have green tens (rods) and yellow ones.

• They will then use jottings of base 10 (sticks and circles) alongside numerals and symbols.



40 + 8 = 48

• Children will move to adding 2 digit numbers and 10s using the steps above, then by grouping the tens together then adding on the ones.
• They will then move to adding two 2 digit numbers using the steps above.

Expanded addition method

This method relies on an understanding of place value therefore children's place value needs to be secure before this is used. Only use this method with numbers that involve 'carrying', otherwise it could be done mentally. Children will be adding numbers by their place value eg. 20 + 50

127+259 100 + 20 + 7
<u>200 + 50 + 9</u>
300 + 70 + 16 = 386

Compact addition method

This method relies on children understanding the difference in place value and column value. Children should only move to this method when their understanding of the expanded method is secure for increasingly larger numbers and longer strings of digits. With compact addition, children will be adding numbers by their column value eg. 4 tens + 3 tens. They will learn how and why we carry digits over into the next column. These carried digits will be stored **below** the answer box.

Carrying from the units column.

	2	4	7
+	1	3	5
	3	8	2
		1	

Carrying from the tens column

	3	3	1
+	2	8	2
	6	1	3
	1		

Carrying from the units and the tens columns

	3	7	9
+	2	5	4
	6	3	3
	1	1	

Carrying into a new column

		9	6	5
	+	2	5	1
	1	2	1	6
			1	

Adding 3 integers (whole numbers) and carrying in 2 columns

		5	2
+	4	1	7
		3	4
	5	0	3
	1	1	

Carrying more than ‘1 ten’ or ‘1 hundred’

```
       7   5
  + 2  8   8
    5  6   9
  -----------
    9  3   2
  -----------
     2    2
```

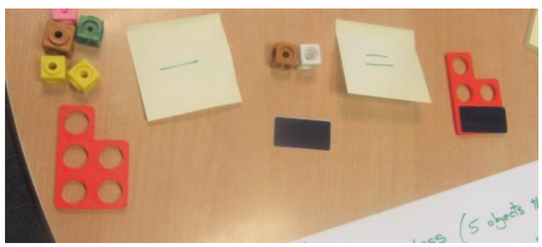
This method will then be used for increasingly larger numbers, longer strings of integers and decimals.

Subtraction

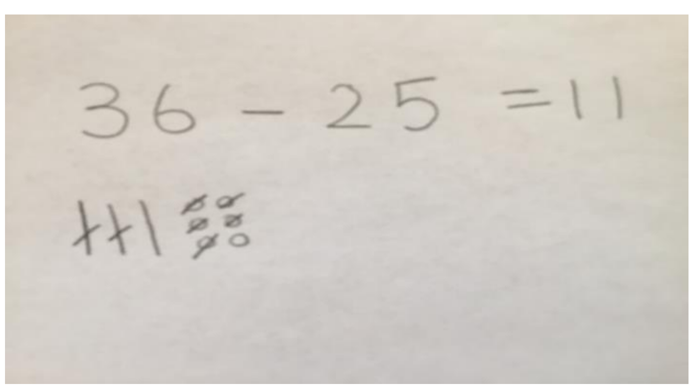
• Using objects, children will find 1 less of numbers up to 5, then up to 10 by physically removing 1 and 1:1 counting what is left.
• As above, taking off more than 1 (2 less, 3 less…)
• Children will use Numicon plates to 10 and will take off numbers using the subtraction covers. They will use the structure of the Numicon plate to find out what is left. (No use of 1:1 counting).

• Children will use objects organised in the Numicon structure, Numicon plates and subtraction covers to subtract:

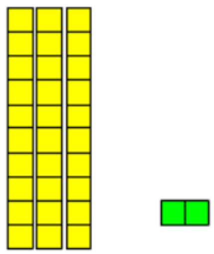
- One-digit and two-digit numbers to 20, including zero.
- A two-digit number and ones.
- A two-digit number and tens.
- 2 two-digit numbers.



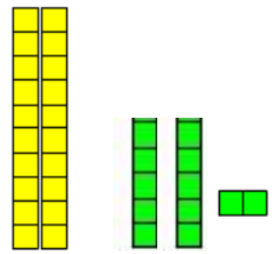
• Children will physically subtract 2 two-digit numbers using Base Ten, where no exchanging is required. They make the first number (subtrahend) from base 10 and take off the second number (minuend) seeing what is left (the difference).
• They will then begin to use Base Ten jottings in books, crossing off the rods/dots that are being taken away (with no exchanging).



• Then children will physically subtract 2 two-digit numbers, where exchanging is required, using Base Ten and exchange a ten rod for 10 ones. Eg. 32 – 24

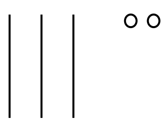


becomes →

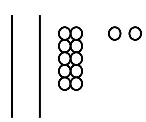


leaving a difference of 8 when the 24 is subtracted

• They will then use base 10 jottings in books for calculations that require exchanging. The 10 rod will be transformed into 10 ones (in the Numicon structure). Eg. 32-24



becomes →



leaving a difference of 8 when the 24 is subtracted

Using their jottings, children will then cross off the number they are taking away (2 tens and 4 ones), leaving a difference of 8.

When children are ready to move to a more formal written calculation, if the digits in the first number (subtrahend) are ALL larger than the digits in the number that is being taken away (minuend) children should be encouraged to work mentally.

Such as in these examples 34 – 21
789 – 362
2956 - 1835

Expanded subtraction method

In this method, the two numbers are partitioned in a way that will allow the subtraction to take place. It is an expanded method leading to decomposition using concrete apparatus of Base Ten from the previous method to support understanding.

Eg. 43 - 27 = 16

	30 ~~40~~	+	1 3	
-	20	+	7	
	10	+	6	

→

	30	13	
-	20	7	
	10	6	= 16

Compact subtraction method

Compact column subtraction involving decomposition

Exchange in the tens column.

$$\begin{array}{ccc} 2 & {}^{6}\cancel{7} & {}^{1}2 \\ & 5 & 7 \\ \hline 2 & 1 & 5 \\ \hline \end{array}$$

Exchange in the hundreds column.

$$\begin{array}{ccc} {}^{2}\cancel{3} & {}^{1}2 & 7 \\ & 5 & 2 \\ \hline 2 & 7 & 5 \\ \hline \end{array}$$

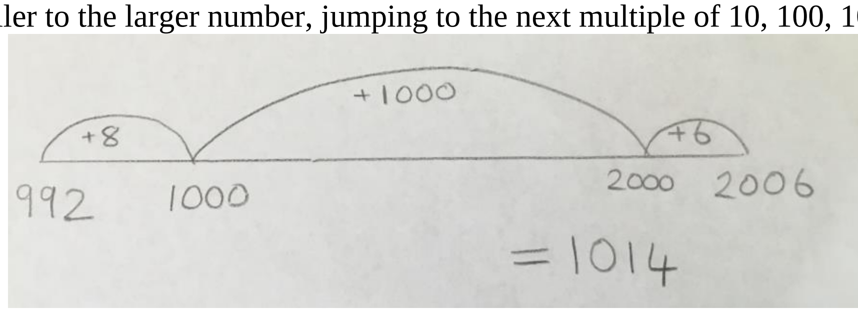
Exchange in both columns.

$$\begin{array}{ccc} {}^{6}\cancel{7} & {}^{1\,1}\cancel{2} & {}^{1}2 \\ & 5 & 8 \\ \hline 6 & 6 & 4 \\ \hline \end{array}$$

Subtractions with **one zero only**.

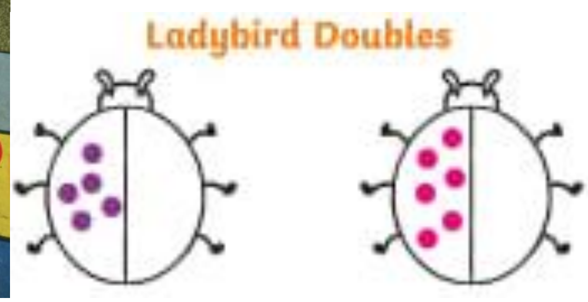
$$\begin{array}{ccc} {}^{5}\cancel{6} & {}^{1}0 & 8 \\ & 3 & 5 \\ \hline 5 & 7 & 3 \\ \hline \end{array}$$

Calculations that include more than one zero eg. 2006 – 992 are best completed with a number line where the child counts up from the smaller to the larger number, jumping to the next multiple of 10, 100, 1000.

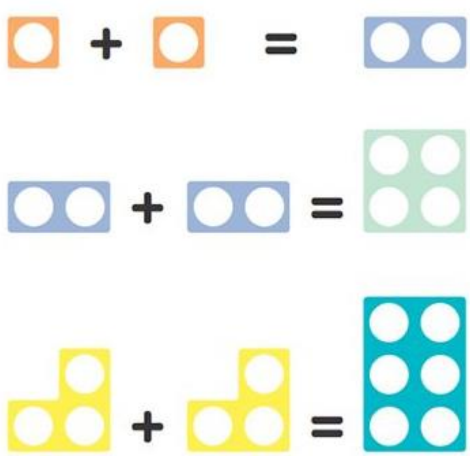


Multiplication

• Children will double (2 lots of) using mirror imaging, having equal groups of physical objects (including fingers, Numicon plates and rekenrek), making sure they are the same. They will total using 1:1 counting, counting on from one of the numbers.
• They will double using equal groups pictorially and total using 1:1 counting, counting on from one of the numbers.

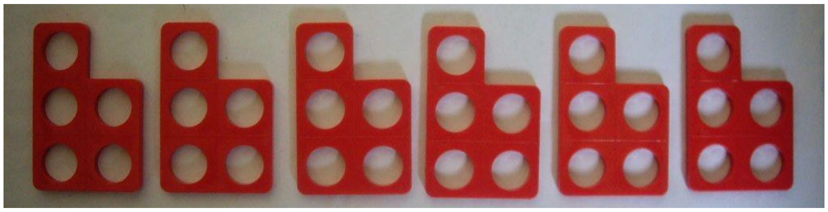


• Using Numicon plates children will double and put the plates together to subitise (know without counting) the total.



• Children use Numicon plates to recall doubles to 20.

• Using Numicon plates, children will find multiples of 2, 5 and 10 by collecting plates multiple times and finding the product either by using the Numicon structure or counting in steps of 2, 5 or 10.

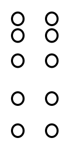
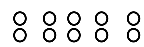



• Using cubes or Numicon pegs (all of the same colour), children will make arrays. Use the arrays to show commutativity (2 x 3 = 3 x 2). They will find the product by counting in steps of 2, 5 or 10.

• Children record multiplications as arrays in their books using circles to represent one. They will use repeated addition or counting in steps of the multiple to calculate answers. Children need to understand that 'multiplied by 6' or 'x 6' means 'add the number six times'.

• Children use digits and symbols to record. They work out answers by drawing arrays and using repeated addition or counting in steps of the multiple to calculate answers (or in some cases, the array will be in a Numicon structure so the product can be subitised).

2 lots of 5	5 lots of 2
2 x 5	5 x 2

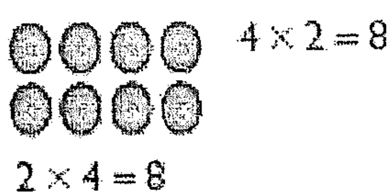



• Extend to multiplying by all numbers up to 10.

Grid methods

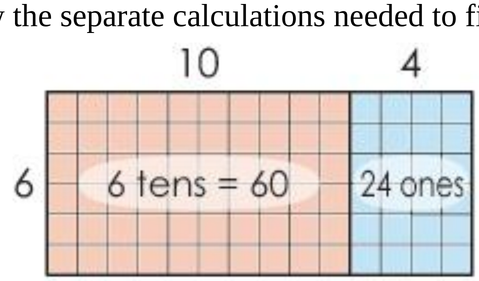
The initial written methods for multiplication build on arrays. Children will use arrays to help them to multiply and prepare them for using the grid method.

Eg.



Using squared paper to draw grids to scale shows children the value of the numbers being multiplied. Partition 2 digit numbers to show the separate calculations needed to find the product.

eg. 14 x 6



60 + 24 = 84

Children then move onto representing the calculation with rectangles in a **closed grid** that is not drawn to scale. The closed grid reflects the calculation. Eg. 2 digit x 1 digit has a 2 x 1 grid

Using the grid method for multiplying a 2/3-digit number by a single digit number.

eg. 13 x 4

	10	3
4	40	12

= 52

Using the grid method for multiplying a 2-digit number by a 2/3 digit number.

eg. 33 x 14

	30	3
10	300	30
4	120	12

420 + 42 = 462

Multiplication of decimals by whole numbers and then decimals by decimals using the grid method.

eg. 8 x 6.24 =

x	6	0.2	0.04
8	48	1.6	0.32

= 49.92

Column methods

Children will move onto the vertical methods alongside the grid method so they can see how the numbers in the vertical method are arrived at.

eg. 13 x 4

	10	3
4	40	12

= 52

$$\begin{array}{rrr} & 1 & 3 \\ \times & & \underline{4} \\ & 1 & 2 \\ & \underline{4} & \underline{0} \\ & \underline{5} & \underline{2} \end{array} \qquad \begin{array}{rrr} & 1 & 3 \\ \times & & \underline{4} \\ & \underline{5} & \underline{2} \\ & {\scriptstyle 1} & \end{array}$$

eg. 33 x 14

	30	3
10	300	30
4	120	12

420 + 42 = 462

$$\begin{array}{rrrr} & & 3 & 3 \\ \times & & \underline{1} & \underline{4} \\ & & 1 & 2 \\ & 1 & 2 & 0 \\ & & 3 & 0 \\ & \underline{3} & \underline{0} & \underline{0} \\ & \underline{4} & \underline{6} & \underline{2} \end{array} \qquad \begin{array}{rrrr} & & 3 & 3 \\ \times & & \underline{1} & \underline{4} \\ & 1 & 3 & 2 \\ & & {\scriptstyle 1} & \\ & \underline{3} & \underline{3} & \underline{\mathbf{0}} \\ & \underline{4} & \underline{6} & \underline{2} \end{array}$$

In this final method children need to cope with multiple calculations in a limited space and follow rules (eg. putting down the 0 before multiplying by the tens number).
This method will not be useful with all children.

It must be noted that there is a change in language between the grid method and the column methods. The grid method is underpinned by place value, therefore the numbers have ‘quantity value’ (where 33 is interpreted as thirty plus three). This is different from the ‘column value’ which underpins the column method (where 33 is interpreted as three in the tens column and three in the units column).

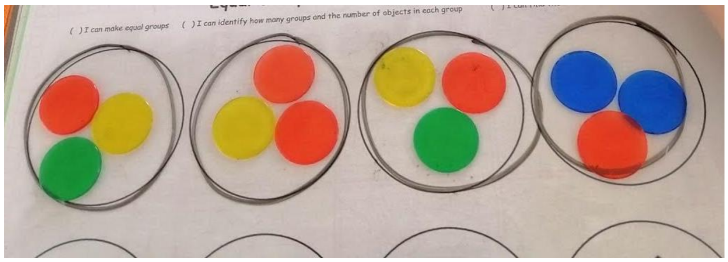
Division

Division as sharing

• Using cubes or other objects, physically share into 2 groups (halving)

• Children use Numicon plates and pegs to work out halves to 20 by sharing into 2 groups. They need to then recall halves to 20.

• Share objects into more than 2 groups using containers or hoops.

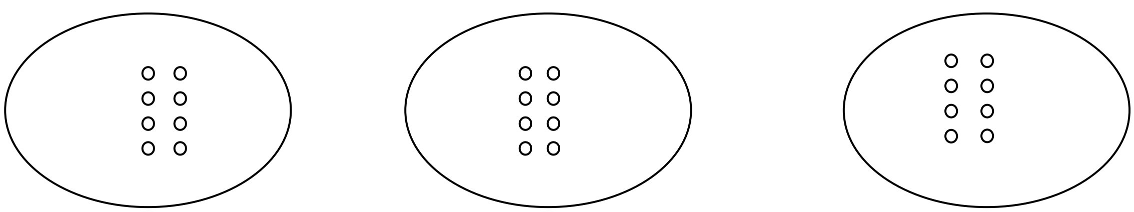


• Share objects into groups of 2, 5 or 10. Make the dividend (number to be divided) using Numicon tiles and fill with pegs, then share into groups.

• Share objects into various numbers of groups as above.

• Children record in books their ones as they share the dividend (drawn as circles to ensure they can be clearly seen), shared into larger circles. Draw the ones in the Numicon structure to subitise (know without counting) the answers.

Eg. 24 ÷ 3 = 8



• Children use digits and symbols to record. Eg. 24 ÷ 3 = 8

Division as grouping

• Children divide by grouping rather than sharing (How many groups of the divisor are there?) They make the number (dividend) with tiles and cover with plates of the divisor

eg. 24 ÷ 3 = 8. Note how the recording looks different for the same calculation.

• Following this, children would then move onto using the Numicon plates alongside the number line below for division (division by grouping).

Number line

Counting on in single jumps of the divisor where there is no remainder. Always see if this can be done mentally first.

39 can be divided into 13 groups of 3 → 39 ÷ 3 = 13

```
 1x3  1x3  1x3   1x3   1x3   1x3  1x3  1x3  1x3 1x3 1x3  1x3  1x3
I-------I-------I-------I-------I-------I-------I-------I-------I------I------I------I------I------I
0       3       6       9       12      15      18      21      24     27     30     33     36   39
```

Counting on in single jumps of the divisor where there is a remainder

10 can be divided into 3 groups of 3 with 1 left over → 10 ÷ 3 = 3 r 1 **(show the remainder as a part jump)**

```
     1x3          1x3          1x3       r1
I-------------I------------I------------I---I-------
0             3            6            9  10
```

Moving towards counting on in multiple steps (chunks) of the divisor. Children first make fact boxes for the divisor to aid them in choosing efficient jumps. It is possible to find the answer from a fact box.

This will be initially without a remainder progressing to calculations which will give a remainder.

e.g. 65 ÷ 5 = 13

```
  2 x 5     2 x 5     2 x 5     2 x 5       2 x 5       2 x 5     1x5
l----------I---------I----------I-----------I-----------I-----------I----------I
0          10        20         30          40          50          60    65   70
```

Fact box
1 x 5= 5
2 x 5 = 10
5 x 5 = 25
10 x 5 = 50

Number line chunking becomes more efficient with larger chunks of the divisor

Fact box
2 x 5 =10
5 x 5= 25
6 x 5 = 30
10 x 5 = 50

e.g. 95 ÷ 5 = 19

```
              10 x 5                              6 x 5           3 x 5
I------------ -------------------------------I-----------------------I-------------I----
0                                            50                      80          95
```

Chunking in a vertical method

Repeated subtraction of chunks of the divisor, firstly without remainders then with remainders. This method maintains the use of the fact box, which if used effectively can get close to the dividend. During this method, children will need to record the chunks of the divisor they have used to calculate the answer later on. This method must only be used if children are confident in column subtraction.

eg. 134 ÷ 4

4) 134
- 120 (30)
14
-12 (3)
2

Fact box
1 x4 = 4
2 x4 = 8
5x4 = 20
10x4 = 40
20 x4 = 80
30 x 4 = 120

Therefore 134 ÷ 4 = 33 r2

This method is to be used to divide by a 2 digit divisor.

24) 5624
- 4800 (200)
824
-720 (30)
104
- 96 (4)
8

Fact box
2 x 24 = 48
4 x 24 = 96
10x24 = 240
5 x 24 = 120
20 x24 = 480
30 x 24 = 720
40 x 24 = 960
100x 24 = 2400
50 x 24 = 1200
200 x 24 = 4800

Therefore 5624 ÷ 24 = 234 r8

Short division

Remember that there is limited mathematical explanation behind this method. Use alongside place value counters to explain the mathematics. Allow children to use it when they are confident in their understanding and explanation of the number line and chunking methods in order to speed up their division by a 1 digit number. If used, ensure children are exposed to varied calculations eg. with zeros in the dividend, with carrying figures that are more than 1.

eg. 291÷ 3 = 97

$$\begin{array}{r|l} & 9\,7 \\ \hline 3 & 2\,9\,^{2}1 \end{array}$$